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Algorithms & Data Structures — Homework 3 — HS 18

Exercise Class (Room & TA): ______________________

Submitted by: ______________________

Peer Feedback by: ______________________

Points: ____

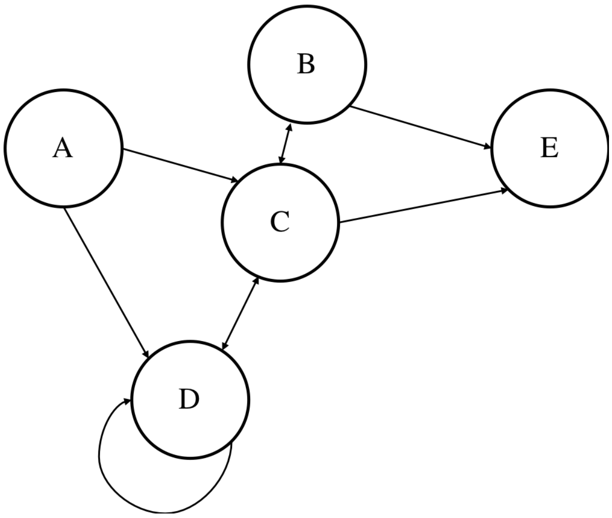
Exercise 3.1 *Big $\mathcal{O}$ Notation* **(1 Point for 4)**.

1. Write the following in Big $\mathcal{O}$ Notation, simplifying them as much as possible.
   - $5n^3 + 40n^2 + 100$
   - $1^2 + 2^2 + 3^2 + \ldots + n^2$
   - $2n \log_3 n^4$
2. Place the following functions in order such that if f appears before g, it means that $f \leq \mathcal{O}(g)$. If multiple functions have the same complexity, please indicate so.

$$2n^2 + n + n^5 \qquad 2.1^n \qquad \log n \qquad n \qquad n \log n \qquad 2^n$$

3. Prove that if $f_1(x), f_2(x) \leq \mathcal{O}(g(x))$, then $f_1(x) + f_2(x) \leq \mathcal{O}(g(x))$. Properties like this are useful with arguing about Big $\mathcal{O}$ Notaaion.
4. Prove or disprove the following.
   - If $f_1(x), f_2(x) \leq \mathcal{O}(g(x))$ then $\frac{f_1(x)}{f_2(x)} \leq \mathcal{O}(1)$. Assume $f_1(x), f_2(x), g(x) > 0$,
   - If $f_1(x) \leq \mathcal{O}(g(x))$ and $f_2(x) \leq \mathcal{O}(\frac{1}{g(x)})$, then $f_1(x)f_2(x) \leq \mathcal{O}(1)$.
     Assume $f_1(x), f_2(x), g(x) > 0$,

Exercise 3.2 *Graph Representation.*



1. For the given graph, express it as an adjacency matrix, and as adjacency lists.
2. For the given adjacency matrix, draw the graph that it represents, and also express it as adjacency lists.

0	1	1	0	0
0	0	0	0	0
1	1	0	0	0
0	0	0	1	1
0	0	0	1	1

3. For the given adjacency lists, draw the graph that it represents, and also express it as an adjacency matrix.

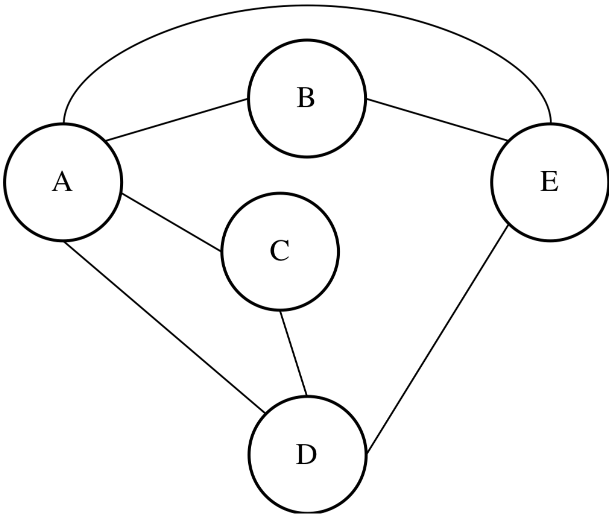
   $A : [D]$
   $B : [C, E]$
   $C : [B]$
   $D : [A, E]$
   $E : [B, D]$

4. For the graphs in 3.2, answer the following:
   - Is it directed or undirected?
   - Is it a bipartite graph?
   - Does the graph contain any loops?
   - Does the graph contain any Euler cycles?
   - Does the graph contain any Euler paths?

Exercise 3.3 *Graph Coloring* **(1 Point for 3, 1 Point for 4)**.

For this exercise, consider the greedy algorithm for graph coloring, where we assign a color to each vertex, and each vertex is assigned a different color than its neighbors. In the algorithm, we traverse the vertices in some order. For each vertex, inspect the color assigned to each adjacent vertex, taking note of which colors are already used. Finally, assign the first free color to the current vertex. If there are no free colors, add a new color to the list.

1. Color the following graph using the greedy algorithm, visiting the vertices in the following order A, B, C, D, E. How many colors did you use? What is the minimum number of colors necessary to color the graph?



2. Construct an undirected graph that, when visiting the vertices in some particular order, the greedy algorithm uses at least twice as many colors as the optimal coloring Give that ordering of the vertices.
3. Show that if every subgraph of an undirected graph has at least one vertex with degree at most k, then the graph can be colored with at most k+1 colors. Hint: use induction on the number of vertices in the graph.
4. Suppose that every subgraph of an undirected graph $G = (V, E)$ has at least one vertex with degree at most k. Provide an algorithm to color G with at most $k + 1$ colors.